Optimization of Resonant Frequency Measurement Algorithm for Wireless Passive SAW Sensors

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***Abstract* — The paper is devoted to reduction of random errors in the wireless measurement of the resonant frequency of a SAW resonant sensor. It is achieved by using weighted averaging of multiple measurement results where the weights depend on the strength of the SAW responses picked up by the interrogation receiver. Optimum weights are found either by analytical method based on statistical information collected for a wireless tire pressure monitoring system (TPMS) or by means of genetic optimization algorithm using numerical model of the sensor. The optimized weights allow reduction of the time needed for averaging by at least a factor of two in comparison with simple averaging required to achieve the same standard deviation of the measured frequency.**

I. INTRODUCTION

Passive wireless sensors based on SAW resonators have been used to measure temperature [1, 2], pressure [3, 4] and torque [5] in a number of industrial and medical applications. Their operation is based on variation of the SAW resonant frequency with temperature and strain. The resonant frequency is measured wirelessly using either a CW signal with a variable frequency or an RF pulse as an interrogation signal. The latter is more suitable for a relatively large separation (around 1 m) between the sensor antenna and the interrogator antenna since the SAW response in the form of a decaying natural oscillation can be easily separated from the interrogation pulse in the time domain. The interrogator can determine the resonant frequency either by measuring the amplitude of the SAW response as a function of the interrogation frequency [6] or by measuring the frequency of the natural oscillation [1, 4].

Whatever interrogation method is used, the measured resonant frequency is affected by the receiver noise introducing random errors in the measurement result thereby limiting resolution and accuracy of the sensing system. The usual approach to improving resolution and accuracy is to repeat the measurement N times and find the average frequency value [1, 4] thus reducing the standard deviation of the measured frequency σ_f by a factor of $N^{1/2}$. This approach works very well for wired measurements or for wireless measurements under stationary conditions when the magnitude of the SAW response stays constant. If the wireless measurement is performed in a dynamic environment then the magnitude of the SAW response may vary significantly during the averaging cycle. The example of such an environment is the tire pressure and temperature measurement system (TPMS) in a car employing the SAW resonant sensors [4]. One averaging cycle in the system lasts for 300…500 ms which may include several rotations of the wheel each causing strong variation of the SAW response magnitude received by the interrogator and affecting statistical properties of the frequency readings. Since the interrogator measures the magnitude of the SAW response anyway, the information about it can be used to improve performance of the system. One possibility is to optimize the averaging in such a way that a minimum standard deviation of the measured frequency is achieved for a fixed number of averaged readings N. Another possible goal is to achieve minimum number of readings N required to reach a given standard deviation of the averaged frequency.

The aim of the paper is to propose an optimized method of averaging with improved performance that takes into account peculiarities of the wireless resonant frequency measurement in TPMS and similar dynamic measurement systems. Section I of the paper describes properties of the system under consideration, section II presents analytical approaches to optimization aimed at achieving either minimum standard deviation of the frequency or minimum measurement time. An alternative approach based on genetic algorithm is described in Section III. Experimental results and conclusions are presented in Sections IV and V.

II. PROPERTIES OF THE WIRELESS SENSOR

Since TPMS is a very good example of a wireless measurement system operating in a dynamic environment (see Fig. 1) it has been used to demonstrate our optimization method. The system described in detail in [4] employs a

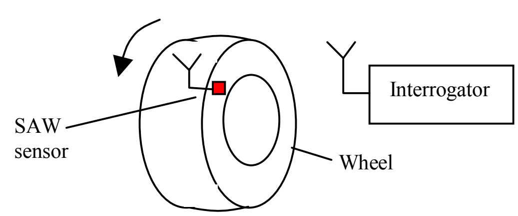


Figure 1. Wireless TPMS based on the resonant SAW sensor

pressure and temperature sensing element based on three SAW one-port resonators connected in parallel to a single antenna. The sensor works in the 433 MHz ISM band. All three resonant frequencies are different and depend on pressure and temperature differently thus allowing calculation of both pressure and temperature from the measured frequency values. Each resonant frequency is measured wirelessly by analyzing the spectrum of the natural oscillation excited in the resonator by a short RF interrogation pulse and picked up by the interrogator's receiver. Coherent accumulation of 20 SAW responses is used to improve the system performance.

Resolution of the sensor is limited by random errors in the wirelessly measured frequencies. Their standard deviation can be estimated as [7,8]

$$\sigma_f = (\sigma_{pn}^2 + \sigma_a^2)^{1/2} \qquad (1)$$

where $\sigma_{pn} \cong 80$ Hz is a contribution due to the phase noise of the receiver's local oscillator and $\sigma_a = C/E_{in}^{-1/2}$ is a contribution due to the receiver's additive noise (C is a constant depending on the receiver properties, E_{in} is the maximum energy spectral density of the input SAW response). The second contribution usually prevails in TPMS so σ_f strongly depends on the magnitude of the received SAW response and hence the angular position of the wheel where the sensor is installed.

Variation of the standard deviation of the measured resonant frequency σ_f with the received SAW response magnitude E_{in} (in arbitrary units) is shown in Fig. 2. As a result of the wheel rotation, the magnitude E_{in} can take any value from approximately 5 dB to 37 dB. The readings with the magnitude smaller than 17 dB are disregarded as unreliable. Distribution of the valid readings among 10 classes, each corresponding to the interval of 2 dB, is shown in Fig. 2 for a particular installation. The data presented in it is sufficient for optimization of the averaging algorithm.

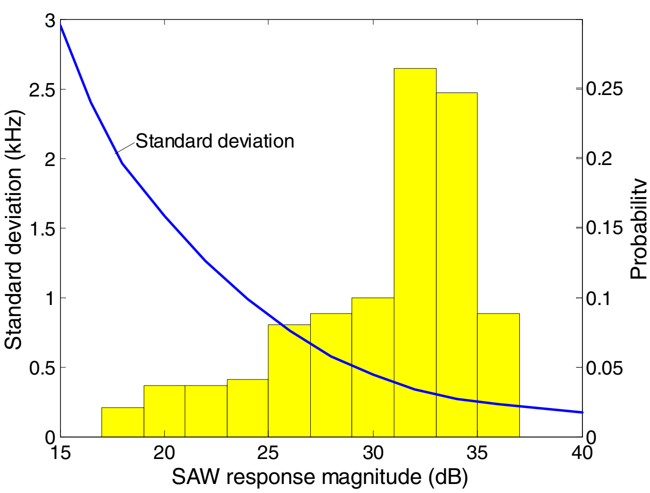


Figure 2. Standard deviation of the measured resonant frequency against the SAW response magnitude and distribution of readings among 10 classes.

III. WEIGHTED AVERAGING OF THE FREQUENCY

A traditional averaging of the measurement results is based on the assumption that σ_f does not vary with time. In the case of a wireless measurement in dynamic environment it is not true any more since σ_f depends on the variable signal-to-noise ratio or the SAW response magnitude E_{in}. As a result each individual valid frequency reading f_j has to be characterised by its own standard deviation σ_j depending on E_{in}. The readings f_j obtained from the SAW responses with higher E_{in} values are more reliable and are characterised by a smaller value of σ_j. Thus, it would make sense to use a weighted average

$$f_{ave} = \frac{\sum_{j=1}^{N} w_j f_j}{\sum_{j=1}^{N} w_j} \qquad (2)$$

instead of a simple one to estimate the value of the wirelessly measured resonant frequency f_0. Here w_j is the weight assigned to the reading f_j depending on the response magnitude and N is the number of different readings taking part in the averaging process. Selection of the optimum weights w_j can be performed in a number of ways.

A. Minimization of the frequency standard deviation

One possible way is to fix the number of valid readings N for each measured resonant frequency and select the weights in such a way that they minimize the standard deviation of the measured average frequency f_{ave}. Assuming that the frequency readings are statistically independent the variance of the average frequency defined by (2) is

$$\sigma_{f\,ave}^2 = \frac{\sum_{j=1}^{N} w_j^2 \sigma_j^2}{\left(\sum_{j=1}^{N} w_j\right)^2}, \qquad (3)$$

and its minimization gives the following optimum weights:

$$w_k = \frac{1}{\sigma_k^2} \frac{\sum_{j=1}^{N} w_j^2 \sigma_j^2}{\sum_{j=1}^{N} w_j} = S / \sigma_k^2, \quad k = 1 \ldots N \qquad (4)$$

where S can have any value. The interrogator measures not only f_j but also $E_{in\,j}$, so using the curve presented in Fig. 2 it can easily determine the optimum weight w_j for each reading. The weight grows with the SAW response magnitude but does not depend on statistical properties of the sensing system. However, the probability distribution for the measured response magnitudes affects the amount of reduction of $\sigma_{f\,ave}$ achieved when using (2) instead of simple average.

To estimate this reduction let us assume that the whole range of E_{in} from the threshold to the maximum observable value is split into K classes (bins) and each class is characterized by its own probability p_m (m = 1…K) and its own standard deviation σ_m. For the data presented in Fig. 2, the threshold is 17 dB, the maximum value is 37 dB and K = 10. Let us also assume that, among N readings, N_m readings belong to the m-th class, which is assigned the optimum weight w_m according to (4). In average $\langle N_m \rangle = p_m N$ where <…> means averaging over a number of attempts to estimate the resonant frequency. Replacing summation over N readings by summation over K classes one can rewrite (2) as

$$f_{ave} = \left(\sum_{m=1}^{K} w_m \sum_{i=1}^{Nm} f_i^{(m)} \right) \Bigg/ \left(\sum_{m=1}^{K} w_m N_m \right) \tag{5}$$

where $f_i^{(m)}$ is the i-th reading (i = 1…N_m) belonging to the m-th class. Similarly, (3) can be rewritten as

$$\sigma_{f\,ave}^2 = \frac{1}{N} \left(\sum_{m=1}^{K} w_m^2 \sigma_m^2 p_m \right) \Bigg/ \left(\sum_{m=1}^{K} p_m w_m \right)^2 . \tag{6}$$

Using (6) for N = 40 and the data presented in Fig. 2 one can obtain the standard deviation of the measured frequency $\sigma_{f\,ave}$ = 101.6 Hz in the case if w_m = 1 and $\sigma_{f\,ave}$ = 54.6 Hz when the optimum weights defined by (4) are used. As one can see, using the optimum weights reduces random errors in the frequency estimate almost by a factor of two.

In practice, obtaining one frequency reading may take up to 1 ms so obtaining one resonant frequency estimate for three resonators and four wheels of the vehicle may take up to 480 ms for N = 40. Bearing in mind that not all the readings taken are valid, the pressure and temperature update period can increase up to approximately 1 s. This update period may be too long for certain applications, in particular, in motor sport. In this case, the weight optimization problem can be formulated in a different way.

B. Minimization of the frequency measurement time

Let us assume that the length of the averaging buffer in the interrogator's processor is fixed to N_a and select the weights in such a way that they minimize time needed to fill in this buffer keeping the same value of the of the standard deviation of the measured averaged frequency. This method allows minimization of the data update period. A version of this method with integer weights $w_j \geq 1$ is especially attractive because it greatly simplifies calculation of (2) since multiplication by w_j can be replaced by writing the frequency value into the buffer w_j times. Obviously, in this case

$$N_{\mathrm{a}} = \sum_{j=1}^{N} w_j \tag{7}$$

and the required number of valid readings N needed to fill in the buffer becomes random. Its mean value

$$\langle N \rangle = N_a / w_{ave} \tag{8}$$

can be considerably smaller than N_{a} leading to a reduction of the measurement time and the data update period since the average weight

$$w_{ave} = \sum_{m=1}^{K} w_m p_m \tag{9}$$

exceeds unity. The following method of calculation of the optimal weights can be used in this case.

The whole range of SAW response magnitudes should be split into K classes the same way as it was described in sub-section A. Taking (6) to (9) into account one can obtain the standard deviation of the average frequency in the form

$$\sigma_{f\,ave}^2 = \frac{1}{N_a} \left(\sum_{m=1}^{K} w_m^2 \sigma_m^2 p_m \right) \Bigg/ \left(\sum_{m=1}^{K} p_m w_m \right). \tag{10}$$

The optimum weights w_m for each of K classes could be found by minimization of (10) as a goal function. However, the result in this case would be trivial – obviously, the minimal value of $\sigma_{f\,ave}$ would be achieved for minimal possible weights w_m = 1. Clearly, it is not a satisfactory result because it does not reduce the measurement time.

A better result can be obtained if the goal function G also includes a term proportional to the time needed to fill in the averaging buffer:

$$G(W) = \sigma^2_{f\,ave} + A/w_{ave} \tag{11}$$

where A is a constant establishing a relative importance of minimizing the buffer filling time and W is the vector of the weights. By varying the value of A one can trade off between the value of $\sigma_{f\,ave}$ characterizing random measurement errors and the average weight value w_{ave} characterizing the length of measurements.

Unlike the weights determined in the previous sub-section, the weights obtained by minimization of the goal function G will depend on statistical properties of the wireless sensing system. Indeed, in the case of a sharp distribution of E_{in}, the weights of the classes with a high probability will be emphasized in comparison with those with a low probability in order to minimize the buffer filling time (or maximize w_{ave}).

Fig. 3 presents the weights optimized using the goal function described by (11) for different values of the parameter A expressed in the units of kHz^2. Statistical data are taken again from Fig. 2 and the buffer length N_a = 40. As one can see, the weights do equal unity if A = 0.001, the smallest value meaning that reduction of the buffer filling time is less significant than reduction of the standard deviation of the average frequency. The larger the value of A, the larger the weights since they accelerate filling in the

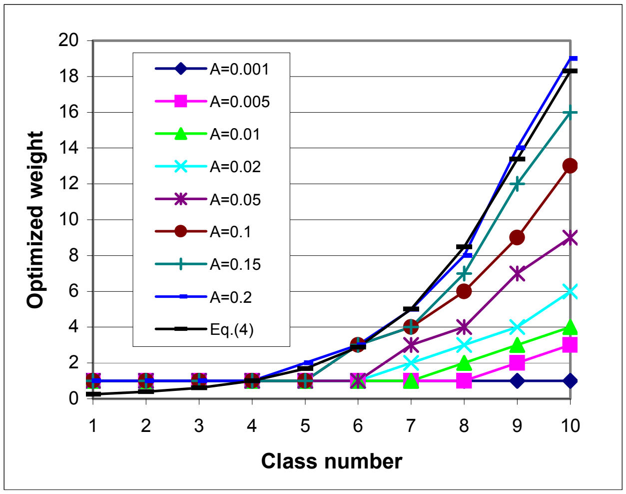
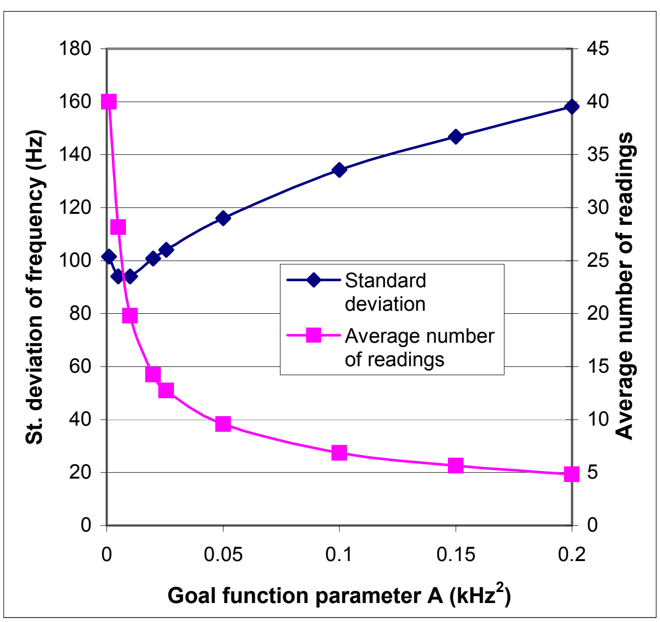


Figure 3. Optimized weights found by minimization of the goal function for different parameters A and by using Eq. (4).

averaging buffer. A general variation of the weights with the class number is similar to the one described by (4) (see the black curve in Fig. 3).

A graph presented in Fig. 4 gives an answer to the question how to select the value of A. It shows variation of the standard deviation of the average frequency and the mean number of readings required to fill in the buffer (calculated using (8)) with the parameter A. When A is large thus giving large weights for the classes corresponding to strong SAW responses, the mean number of readings required for averaging may be reduced by a factor of eight but the optimum achievable value of $\sigma_{f\,ave}$ will be 60% higher than the un-optimized one. If the value of A is selected in such a way that both terms of the goal function $G(W)$ make



comparable contributions then one can expect to achieve a compromise between the amount of random errors in the average measured frequency and the number of required readings. Selecting

$$A \approx \left(\sum_{m=1}^{K} \sigma_m^2 \right) \Big/ (KN_a) \tag{12}$$

gives the value A = 0.0256 kHz^2. As one can see from Fig. 4 the value of $\sigma_{f\,ave}$ in this case is just slightly above the un-optimized value. If A = 0.02 kHz^2 then the standard deviation stays the same but the mean number of the required readings is reduced from 40 to 14.2, i.e. by a factor of 2.8.

IV. Genetic Algorithm for Optimization of the Weights

Results presented in section III were obtained by means of finding the minimum of the goal function $G(W)$ regarding it as a function of real arguments with a subsequent rounding W to the integer values $w_j \geq 1$. Optimization was based on the previously evaluated statistical performance of the sensor that depends on the installation. In the case of TPMS, variation of the SAW response magnitude E_{in} with the rotation angle may be quite different for all four wheels [4]. As a result the distribution of E_{in} may also differ from one wheel to another which can potentially cause difference in the optimum weights. Although the optimization can be done for each vehicle platform individually it would be desirable, especially for aftermarket installations, if the system could optimize the weights itself during the initial, "learning" phase of its operation. This could be done using a genetic algorithm.

The essence of the method is in the evaluation of the performance of several sets of the averaging weights and iteratively doing the following steps:

- selecting a certain number of the best performing sets (children) and allowing them to live on into the next generation (as parents),
- introducing random mutations into the strongest performers and also adding them to the next generation,
- randomly pairing the strongest contenders and crossing over randomly selected sections of their weights to create further children.

Each generation consists of 12 sets, each set containing K = 10 weights. Their performance is evaluated using the goal function

$$G'(W) = \sigma_{F\,ave}/\sigma_T + B(\langle N \rangle / N_a) \tag{13}$$

analogous to the one described by (11) where $\sigma_{F\,ave}$ is the standard deviation of the difference between the resonant frequencies of the two SAW resonators, N_a = 40, $\sigma_T \cong$ 95 Hz is the target standard deviation, i.e. the one that is achieved if all the weights equal unity, and B is a constant establishing a relative importance of minimizing the buffer filling time.

The algorithm has been tested using the data obtained from computer simulation of the TPMS rather than from a running vehicle. The stochastic computer model was based on real system timing and a real variation of the SAW response magnitude with the wheel rotation angle (shown in Fig. 5) for the installation on a passenger car different from the one described in Sections II and III.

Table 1 shows key iterations/generations for B = 3.5, arriving at the optimum weights for a minimum number of readings, while maintaining a standard deviation close to σ_T.

TABLE I. OPTIMIZATION OF WEIGHTS FOR $B = 3.5$

Generation number	σ_{Fave} (Hz)	Mean number of readings $\langle N \rangle$	Weights w_m
0	97.0	40	1 1 1 1 1 1 1 1 1 1
1	93.8	23	1 1 1 1 1 2 2 2 2 2
2	106.4	19	1 1 2 1 1 2 1 3 1 3
3	89.1	15	1 1 1 2 1 2 2 3 2 4
7	113.9	10	1 1 1 2 2 6 1 4 4 7
10	93.3	9	1 1 3 1 1 2 4 3 6 10
13	87.7	10	1 1 1 1 1 2 2 1 6 9
30	86.9	9	1 1 1 1 1 1 1 1 7 10
38	99.0	8	1 1 1 1 1 1 1 3 9 11
42	**91.0**	**8**	**1 1 1 1 1 1 2 4 7 11**

As one can see from Table 1, the algorithm converges to the optimum set of weights after 42 generations. The set allows filling in the averaging buffer with only 8 readings instead of 40 that reduces the measurement time by a factor of five. The weights are not far from those obtained in Section III for A = 0.05. However, the achieved value of the standard deviation $\sigma_{F\ ave}$ is lower because the magnitudes of the SAW responses for this particular installation are higher than those in Section III by 5 dB (see Figs. 2 and 5).

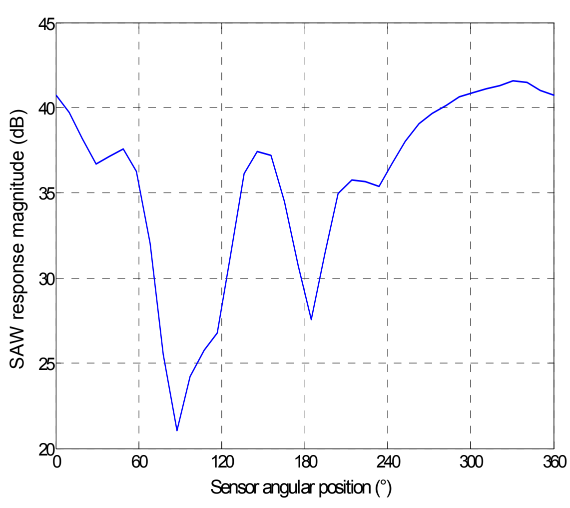


Figure 5. Variation of the SAW response magnitude with the wheel rotation angle used in the computer model for genetic optimization of the weights.

V. EXPERIMENTAL RESULTS AND THEIR DISCUSSION

Experimental verification of the new frequency measurement algorithm was performed for the same TPMS installation as the one used in Section IV for genetic optimization of the averaging weights.

Averaging of the wirelessly measured resonant frequency difference between the two resonators in the sensor was done in the buffer conatining N_a = 40 readings in two cases: with the unity weights and with the weights taken from the last row of Table 1. Results of the averaged difference frequency measurement are presented for both cases in Figs. 6 and 7 against the reading number. The lower graph in Fig. 6 also presents the number of the interrogations N required for filling in the averaging buffer. One can see that using the weighted averaging considerably reduces the number

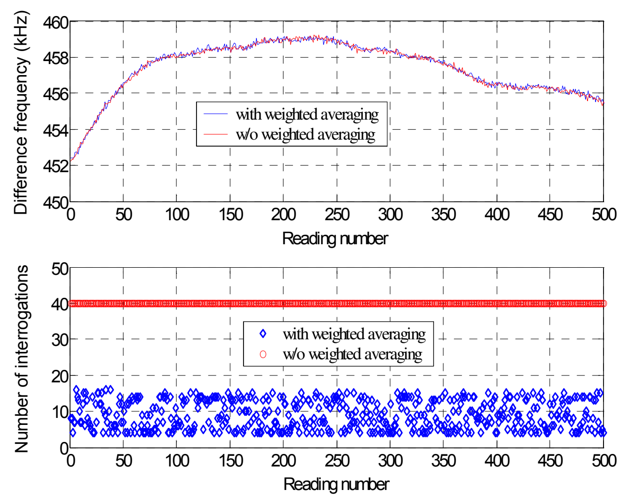


Figure 6. Measured variation with time (reading number) of the averaged difference frequency (upper graph) and the number of readings N required to fill in the averaging buffer (lower gaph).

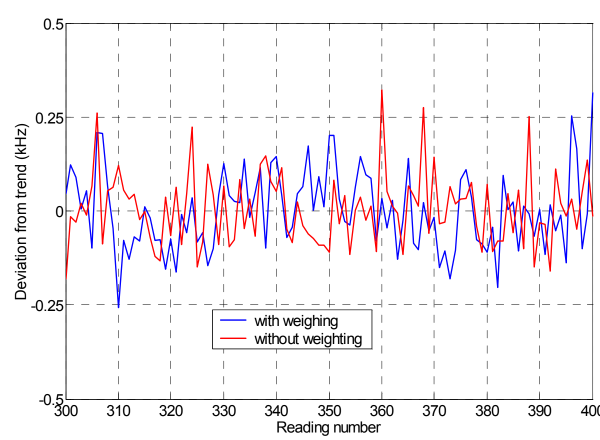


Figure 7. Deviation of the measured average difference frequency from the trend line.

of sensor readings indeed. At the same time, as one can see in Fig. 7 obtained after removing a trend from the frequency measurement results, the amount of random errors remains approximately the same: $\sigma_{F\,ave}$ = 119 Hz for unity weights and $\sigma_{F\,ave}$ = 106 Hz when using weighted averaging.

Fig. 6 shows that the number of readings required for averaging N is a random number varying from 4 to 16 and its mean value is $\langle N \rangle = 9$ which is close to the value predicted in Table 1. In practice, it might be advantageous to output the updated values of pressure and temperature over equal intervals of time rather than at random moments of time. Even selecting a constant number of averaged readings $N = 16$ would ensure reduction of the measurement time by more than a factor of two for a guaranteed maximum amount of random errors.

VI. CONCLUSIONS

A new method of averaging of wirelessly measured resonant frequencies has been proposed for a passive resonant SAW sensor working in a dynamic environment. The method is based on weighted frequency averaging where the weights are selected on the basis of the magnitude of the SAW responses received by the interrogator.

Methods of determining optimum weights have been developed achieving either minimum random errors of the average measured frequency for a given number of readings or a minimum number of readings required to obtain a given standard deviation of the measured frequency. Optimization has been performed using two approaches: (a) minimization of the goal function on the basis previously obtained statistical information on the sensing system and (b) genetic optimization of the weights that can be performed during the "learning phase" of operation of the sensing system.

Experiments with the real tire pressure monitoring system based on a passive resonant SAW sensor installed on a passenger car have confirmed that the weighted averaging algorithm does reduce the mean required number of averaged readings by a factor of 4.4 keeping the same amount of random errors.

Although the method has been developed for wireless resonant sensors it can also be applied to other types of passive wireless sensors, for instance, the sensors based on SAW reflective delay lines.

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